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EVALUATION OF BUCKLING AND FIRST-PLY FAILURE PROBABILITIES OF COMPOSITE LAMINATES

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Abstract—A procedure for failure probability evaluation of composite laminates subjected to in-plane loads is proposed. The material properties, fiber angles and layer thicknesses of the laminates are treated as random variables in the reliability analysis. The statistics of first-ply failure loads and buckling strengths of the laminates are determined via the stochastic finite element method. The failure probabilities of the laminates which are susceptible to buckling and first-ply failure are computed using the statistics obtained in the stochastic finite element analysis. The feasibility and accuracy of the present approach are validated using the results obtained via the Monte-Carlo method. A number of examples of reliability analysis of composite laminates subject to in-plane loads are given to illustrate the applications of the procedure. © 1998 Elsevier Science Ltd.

1. INTRODUCTION

Laminated composite materials have become an important engineering material for the construction of automobile, mechanical, space and marine structures in the past decade. The use of laminated composite materials in designing these structures has resulted in a significant increase in payload, weight reduction, speed, maneuverability and durability. In pursuing these achievements, the reliability design of laminated composite structures has thus become an important subject of research. A number of researchers have studied the failure probability of composite laminates subjected to in-plane loads (Cederbaum *et al.*, 1990; Sun and Yamada, 1978). Cassenti (1984) investigated the failure probability and probabilistic location of failure in composite beams based on the weakest-link hypothesis. Kam and his associates (1992, 1993) and Engelstad and Reddy (1992) studied the reliability of linear or nonlinear laminated composite plates subjected to transverse loads. In the previous reliability analysis of composite laminates, only one failure mode, e.g. first-ply failure or ultimate fracture, was considered. In this paper, a procedure is developed for reliability analysis of laminated composite plates with random material properties and uncertain stacking sequences subject to in-plane loads. Two failure modes, namely, buckling and first-ply failure are considered in the reliability analysis. The stochastic finite element method is used to obtain the statistics of buckling strength and first-ply failure load required for reliability analysis. The reliability assessment of the laminated composite plates is achieved using the strength statistics and the probability theories. The feasibility and applications of the present procedure are demonstrated by means of examples of reliability analysis of laminates subject to in-plane loads. Results obtained from the Monte-Carlo method are used to validate the accuracy of the present procedure.

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2. UNCERTAINTIES IN COMPOSITE LAMINATES

A composite laminate is a stack of layers of fiber-reinforced laminae. The fiber-reinforced laminae are made of fibers and matrix, which are of two different materials. The way in which the fibers and matrix materials are assembled to make a lamina, as well as the layup and curing of laminae are complicated processes and may involve a lot of uncertainty. Therefore, the material properties of a composite laminate are random in nature. In the following stochastic finite element analysis, the elastic moduli (E_1, E_2, ν_{12}, G_{12}, G_{13}, G_{23}) of the material are treated as independent random variables, and their statistics are used to predict the mechanical behavior of composite laminates. Furthermore, fiber orientations and thicknesses of laminae may fluctuate in the vicinity of the prescribed values, depending on the manufacturing process. It is, therefore, necessary and desirable to investigate the effects of the uncertain stacking sequence on the reliability of composite laminates. Herein, the fiber orientation, θ, and the thickness t, of each layer are also considered to be random. The uncertainties of the stacking sequence can be expressed in the following forms (Nakagiri *et al.*, 1986):

$$\theta_i = \bar{\theta}_i(1+\omega_i) \tag{1a}$$

$$t_i = \bar{t}_i(1+\eta_i) \quad i = 1,2,\ldots,N \tag{1b}$$

where ω_i and η_i stand for random variables for θ_i and t_i, respectively; $\bar{\theta}_i$ and $\bar{t}_i$ are the mean values of the variables θ_i and t_i, respectively; N is the number of layers. It is noted that uncertain layer thickness can cause uncertainty in the z-coordinates of the layer boundary and centroid.

From now on, α_i ($i = 1,2,\ldots,2N+6$) will be used to denote the basic random variables in which α_i ($i = 1, 2,\ldots, N$) denote the fiber orientations, α_i ($i = N+1,\ldots,2N$) the layer thicknesses, and α_i ($i = 2N+1,\ldots,2N+6$) the material properties E_1, E_2, ν_{12}, G_{12}, G_{13} and G_{23}, respectively. The afore-mentioned uncertainties in mechanical properties and stacking sequence of composite laminae can cause variations in the elements of the constitutive matrix of the laminate.

3. STOCHASTIC FINITE ELEMENT ANALYSIS

The present stochastic finite element analysis of laminated composite plates consisting of random parameters is based on the first-order shear deformation theory (Mindlin, 1951) and the mean-centered second-order perturbation technique. Spatial variability is not considered in the stochastic finite element formulation. The shear deformable finite element developed by Kam and Chang (1992a, 1992b) is used in the finite element analysis. The element can be applied to the analyses of both thin and thick plates, and it contains five degrees-of-freedom (three displacements and two slopes, i.e. shear rotations) per node. In evaluating the terms of element stiffness matrix, a quadratic element of the serendipity family and the reduced integration are sued. The load–displacement relation of a laminated composite plate can be expressed as

$$\mathbf{KD} = \mathbf{P} \tag{2}$$

where $\mathbf{K}$ is the structural stiffness matrix, $\mathbf{D}$ the vector of nodal displacements, and $\mathbf{P}$ the vector of nodal forces. Detailed derivation of the stochastic finite element method has been reported in the literature (e.g. Kam and Lin, 1992). A brief review of the method is given as follows.

Based on the mean-centered second-order perturbation technique, the stiffness matrix, $\mathbf{K}$, is expanded in terms of the random variables α_i ($i = 1, 2,\ldots, 2N+6$), which represent structural uncertainties existing in the plate, as

$$\mathbf{K} = \mathbf{K}^{(0)} + \sum_{i=1}^{M} \mathbf{K}_{,i}^{(1)}\,\delta\alpha_i + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M} \mathbf{K}_{,ij}^{(2)}\,\delta\alpha_i\delta\alpha_j \tag{3}$$

where $\delta\alpha_i = \alpha_i - \bar{\alpha}_i$ with $\bar{\alpha}_i$ denoting the mean value of the random variable α_i; $M = 2N+6$; $\mathbf{K}^{(0)}$ is the zeroth-order structural stiffness matrix, which is identical to the deterministic structural matrix; $\mathbf{K}_i^{(1)}$ is the first-order structural stiffness matrix with respect to random variables α_i; and $\mathbf{K}_{,ij}^{(2)}$ is the second-order structural stiffness matrix with respect to random variables α_i and α_j. The nodal displacements are also influenced by the structural uncertainties and thus the displacement vector possesses a similar expression:

$$\mathbf{D} = \mathbf{D}^{(0)} + \sum_{i=1}^{M} \mathbf{D}_{,i}^{(1)}\delta\alpha_i + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M} \mathbf{D}_{,ij}^{(2)}\,\delta\alpha_i\delta\alpha_j. \tag{4}$$

4. FIRST-PLY FAILURE LOAD

A composite laminate is assumed to fail when any ply in the laminate fails. Failure of the laminate is determined from first-ply failure analysis in which the Tsai–Wu criterion is adopted. If λ_{p} is defined as the strength ratio, Tsai–Wu criterion (Tsai, 1980) expressed in tensor form can be written as

$$\lambda_{\mathrm{p}}^2 F_{ij}\sigma_i\sigma_j + \lambda_{\mathrm{p}} F_i\sigma_i - 1 = 0 \tag{5}$$

where F_{ij}, F_i are functions of material strengths and σ_i are stresses in material directions. It is noted that failure of the laminate occurs when the strength ratio of any ply, λ_{p}, is less than or equal to the applied load. Again the mean-centered second-order perturbation technique and the stochastic finite element method can be used to find the statistics of strength ratio from eqn (5). The mean and variance of the strength ratio are expressed as

$$E[\lambda_{\mathrm{p}}] = \lambda_{\mathrm{p}}^{(0)} + \frac{1}{2}\sum_{k=1}^{M}\sum_{l=1}^{M} \lambda_{\mathrm{p},kl}^{(2)} E[\delta\alpha_k\delta\alpha_l] \tag{6}$$

and

$$E\left[\left(\lambda_{\mathrm{p}} - \lambda_{\mathrm{p}}^{(0)}\right)^2\right] = \sum_{k=1}^{M}\sum_{l=1}^{M} \lambda_{\mathrm{p},k}^{(1)}\lambda_{\mathrm{p},l}^{(1)} E[\delta\alpha_k\delta\alpha_l]. \tag{7}$$

The zeroth, first, and second-order strength ratios in the above equations can be determined from the truncated Taylor series form of eqn (5) following the same procedure as described in the previous section. It is noted that the layer that possesses the largest failure probability is used to determine the statistics of the first-ply failure load of the laminate in the above analysis.

5. BUCKLING STRENGTH

The deterministic approach to evaluate the buckling load of a composite laminate can be found in the literature (Kam and Chang, 1992b). Herein, the effect of initial imperfections is not considered in the buckling analysis. The buckling load of a laminate is determined by solving the following eigenvalue problem:

$$[\mathbf{K} + \lambda_{\mathrm{b}}\mathbf{K}_{\mathrm{g}}]\delta\mathbf{D} = 0 \tag{8}$$

where λ_{b} is load multiplier and $\mathbf{K}_{\mathrm{g}}$ is the geometrical stiffness matrix of the laminate subject to edge loads of unit magnitude. The smallest value of the load multiplier is defined as the buckling strength of the laminate. The statistics of buckling strength can be determined from the above equation via the mean-centered second-order perturbation technique and the stochastic finite element method. The mean and variance of buckling strength are expressed as

$$E[\lambda_b] = \lambda_b^{(0)} + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}\lambda_{b,ij}^{(2)}E[\delta\alpha_i\delta\alpha_j] \tag{9}$$

and

$$E\left[\left(\lambda_b - \lambda_b^{(0)}\right)^2\right] = \sum_{k=1}^{M}\sum_{l=1}^{M}\lambda_{b,k}^{(1)}\lambda_{b,l}^{(1)}E[\delta\alpha_k\delta\alpha_l]. \tag{10}$$

6. RELIABILITY ANALYSIS

The reliability assessment of a composite structure, in general, requires information on the probability distribution and not just on the statistical moments of the strength of the structure. In the above stochastic finite element analysis of composite laminates, however, only statistical moments of strength ratio and buckling load can be determined while the types of probability distributions of the above strength variables are indeterminate. Therefore, it is worth studying the effects of various probability distributions on laminate reliability before any attempt to choose probability distribution types for the strength variables is made. Let $f_{\lambda_p}(u)$ and $f_{\lambda_b}(v)$ be the probability density functions of strength ratio and buckling load, respectively. For the deterministic applied load, P_C, the failure probabilities of the laminate subject to either first-ply failure or buckling are determined from the following equations:

First-ply failure

$$P_f = \int_0^{P_c} f_{\lambda_p}(u)\,du. \tag{11}$$

buckling failure

$$P_f = \int_0^{P_c} f_{\lambda_b}(v)\,dv. \tag{12}$$

In the following analysis, three types of probability distributions, namely, normal, lognormal and Weibull distributions will be adopted in eqns (11) and (12) to evaluate the failure probabilities of the laminates.

In case the two failure modes are dependent, the correlation between buckling strength and first-ply failure load can be estimated from the covariance of λ_p and λ_b:

$$\mathrm{cov}[\lambda_p, \lambda_b] \cong \sum_{i=1}^{M}\sum_{j=1}^{M}\lambda_{p,i}\lambda_{b,j}\,\mathrm{cov}[\alpha_i, \alpha_j] \tag{13}$$

where cov[·] denotes covariance. It is noted that if the basic random variables α_i are independent, $\mathrm{cov}[\alpha_i, \alpha_j] = 0$ when $i \neq j$ and $\mathrm{cov}[\alpha_i, \alpha_j]$ becomes the variance of α_i when $i = j$. In the above equation both $\lambda_{p,i}$ and $\lambda_{b,j}$ can be determined in the previous sections. The coefficient of correlation for λ_p and λ_b is thus obtained as

$$\rho = \frac{\mathrm{cov}[\lambda_p, \lambda_b]}{\Delta_{\lambda_p}\Delta_{\lambda_b}} \tag{14}$$

where Δ denotes standard deviation. Herein, both λ_p and λ_b are assumed to be lognormal variates. The joint probability density function of λ_p and λ_b is written as

$$f_{X,Y}(x,y) = \frac{1}{2\pi\Delta_X\Delta_Y\sqrt{1-\rho^2}}\exp\left[\frac{-1}{2(1-\rho^2)}\left\{\left(\frac{x-\bar{X}}{\Delta_X}\right)^2 - 2\rho\left(\frac{x-\bar{X}}{\Delta_X}\right)\left(\frac{y-\bar{Y}}{\Delta_Y}\right)+\left(\frac{y-\bar{Y}}{\Delta_Y}\right)^2\right\}\right]\frac{1}{\lambda_b}\frac{1}{\lambda_p} \quad (15)$$

where $X = \ln\lambda_p$ and $Y = \ln\lambda_b$. Define the probabilities of the following failure events as

$$P[C_1] = P[\text{laminate fails due to first-ply failure}] = \int_0^{P_c} f_{\lambda_p}(u)\,du \quad (16a)$$

and

$$P[C_2] = P[\text{laminate fails due to buckling}] = \int_0^{P_c} f_{\lambda_b}(u)\,du. \quad (16b)$$

The reliability of the laminate subject to both first-ply failure and buckling is thus expressed as

$$P_s = 1-P_f = 1-\{P[C_1]+P[C_2]-P[C_1\cap C_2]\} \quad (17)$$

where P_s is reliability; P_f is failure probability. The joint probability in eqn (17) is expressed as

$$P[C_1\cap C_2] = \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{x}\int_{-\infty}^{y}\exp\left[-\frac{u^2-2\rho uv+v^2}{2(1-\rho^2)}\right]du\,dv \quad (18a)$$

with

$$x = \frac{\ln P_c - E[\ln\lambda_p]}{\Delta_{\ln\lambda_p}}$$

$$y = \frac{\ln P_c - E[\ln\lambda_b]}{\Delta_{\ln\lambda_b}} \quad (18b)$$

The probability given by eqn (18) can be evaluated using the BNRDF routine of IMSL mathematical package (1989).

7. EXPERIMENTAL INVESTIGATION

The probability distribution of laminate buckling strength was studied experimentally. A number of *Gr/Ep* $[0^\circ/90^\circ/0^\circ/90^\circ]_{2S}$ square laminates of size 10×10 cm were subjected to axial buckling test using a 10-ton Instron testing machine. The top and bottom edges of the laminates were clamped during test. The test procedure has been reported in the literature (Kam and Chu, 1995). A typical load–stroke relation of the laminate is shown in Fig. 1. The test results were fitted by normal, Weibull or lognormal distributions as shown in Figs 2–4. It is noted that lognormal distribution yields the best fit of the test data. Therefore, it is reasonable to assume lognormal distribution for buckling load in reliability analysis of laminates.

8. RESULTS AND DISCUSSION

The afore-mentioned stochastic finite element method (SFEM) and the reliability evaluation technique are used to study the reliability of the angle-ply laminate in Fig. 5. The statistics of the random parameters used in the analysis are listed in Table 1. The

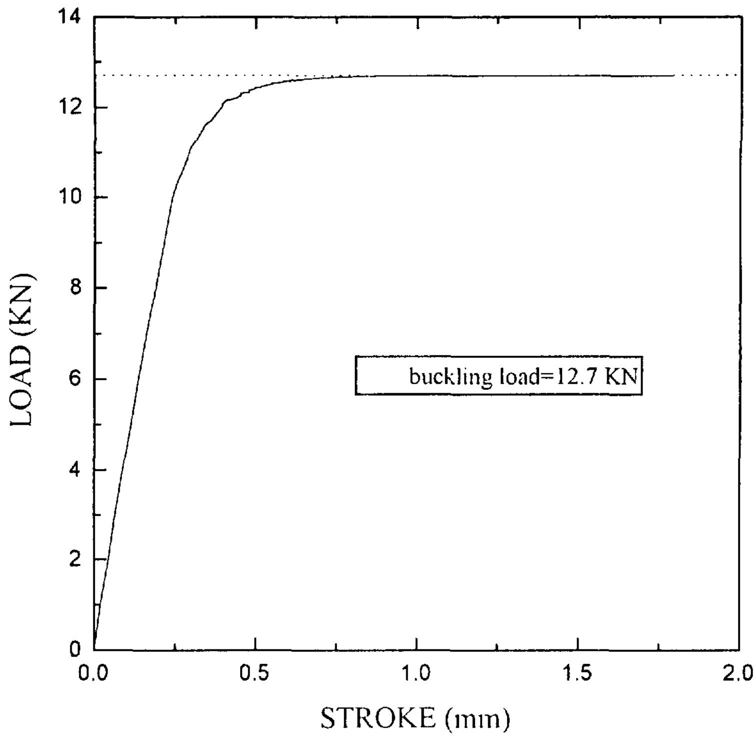

Fig. 1. Load–stroke relation of laminate subject to buckling test.

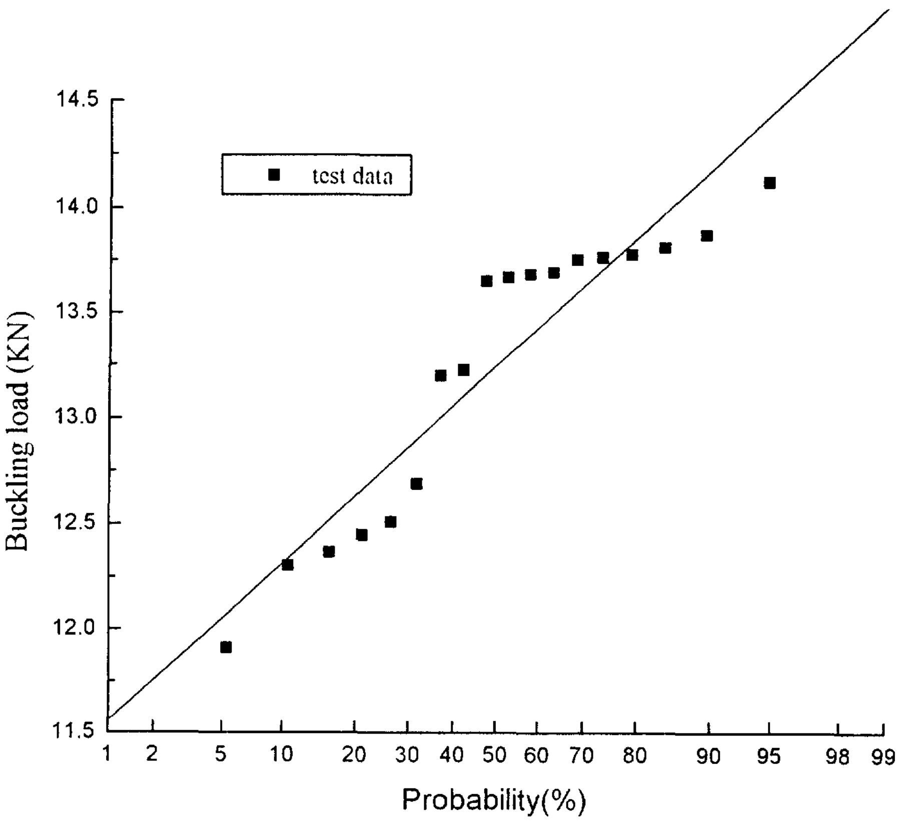

Fig. 2. Buckling test data fitted by normal distribution.

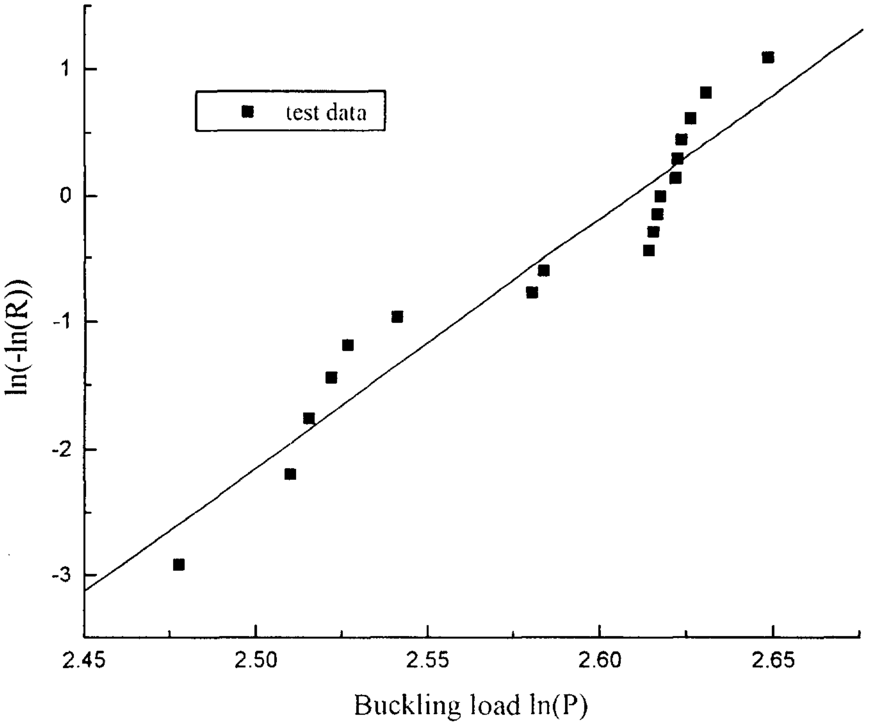

Fig. 3. Buckling test data fitted by Weibull distribution.

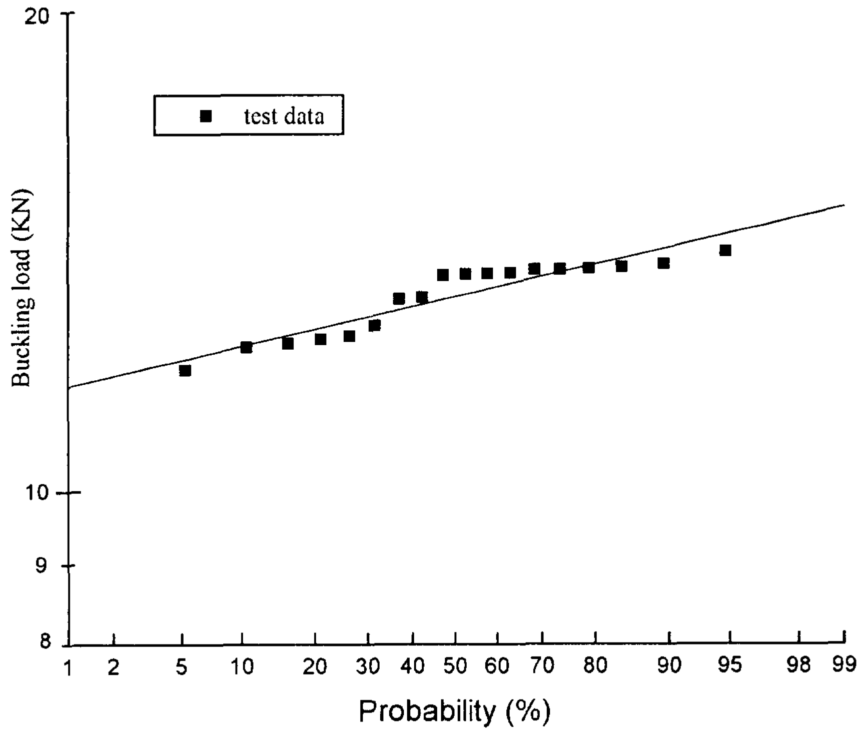

Fig. 4. Buckling test data fitted by lognormal distribution.

laminate is subjected to uniform compressive edge load of 57 lb in^{-1} in X-direction. The mean first-ply failure or buckling loads of the laminate of constant mean thickness with different layup patterns are shown in Fig. 6. It is noted that the number of ply groups has a more significant effect on the mean buckling load than the mean first-ply failure load of the laminate, and the first-ply failure load is much higher than the buckling load for fiber

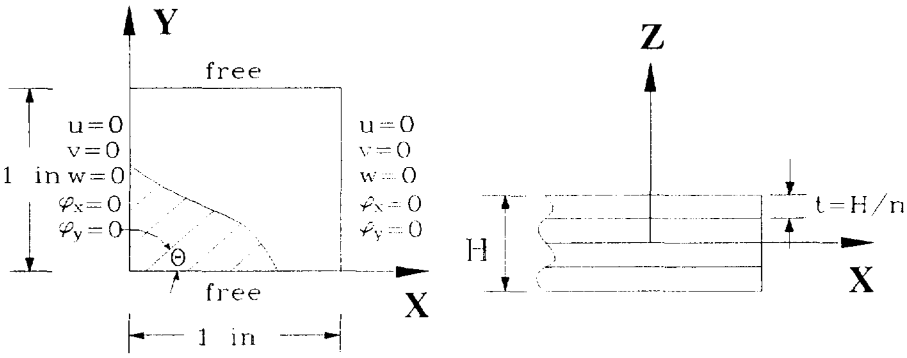

Fig. 5. Geometry and boundary conditions of laminate.

Table 1. Statistics of material properties

Random variable	Expected value	Standard deviation	Material strength	Expected value
E_1	19.2×10^{-6} psi	0.96×10^{-6} psi	X_T	$219.5 \times 10{-}{-}3$
E_2	1.56×10^{-6} psi	0.78×10^{-5} psi	X_C	246.0×10^{-3}
G_{12}	0.82×10^{-6} psi	0.41×10^{-5} psi	Y_T	6.350×10^{-3}
G_{13}	0.82×10^{-6} psi	0.41×10^{-5} psi	Y_C	6.350×10^{-3}
G_{23}	0.49×10^{-6} psi	0.245×10^{-5} psi	R	9.800×10^{-3}
V_{12}	0.24	0.012	S	12.60×10^{-3}
H	0.03 in	0.003 in	T	9.800×10^{-3}
Θ	—	1 or 3°	—	—

* Subscripts T and C denote tension and compression, respectively; S is in-plane shear strength; R, T are transverse shear strengths.

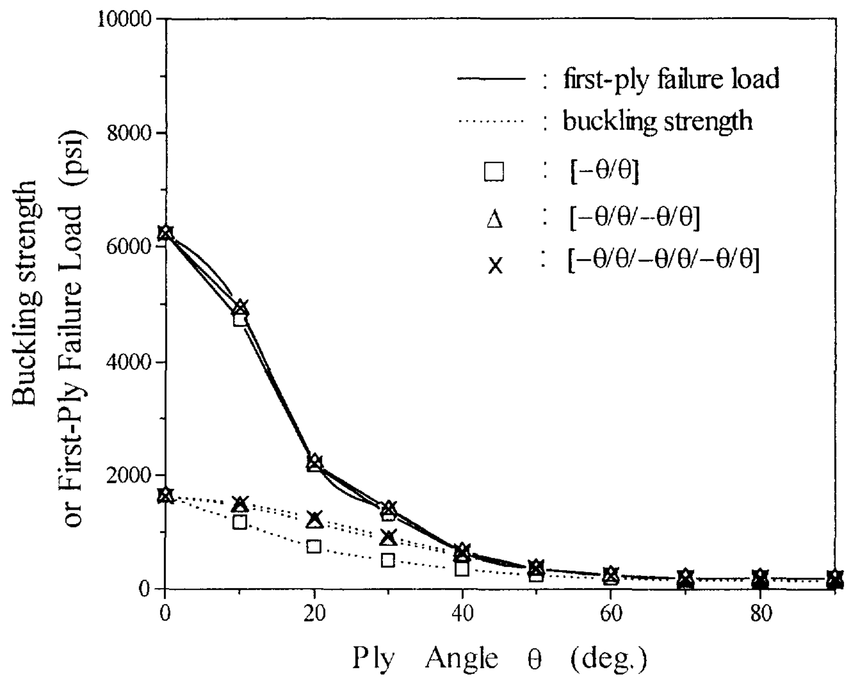

Fig. 6. Expected values of buckling strength and first-ply failure load for laminates of constant thicknesses with different fiber angles and numbers of layers.

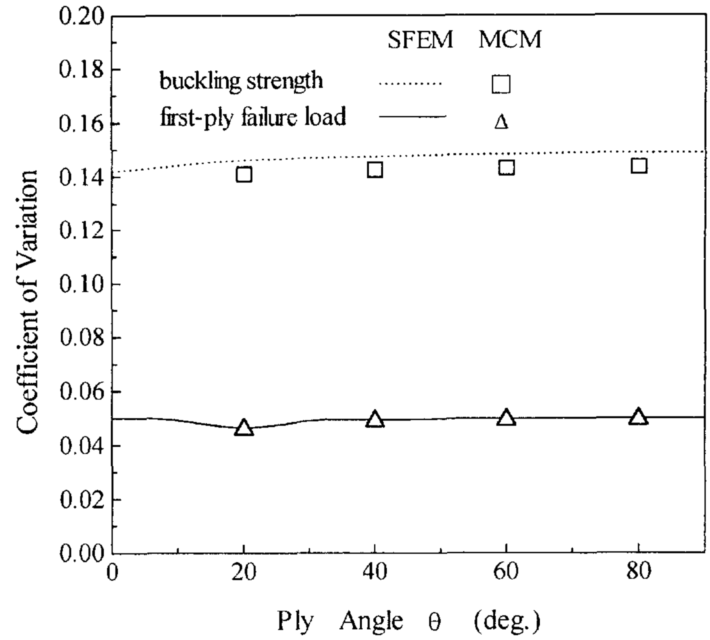

Fig. 7. Coefficients of variation of different types of strength vs fiber angle of a two-layered $[-\Theta/\Theta]$ laminate with random layer thicknesses.

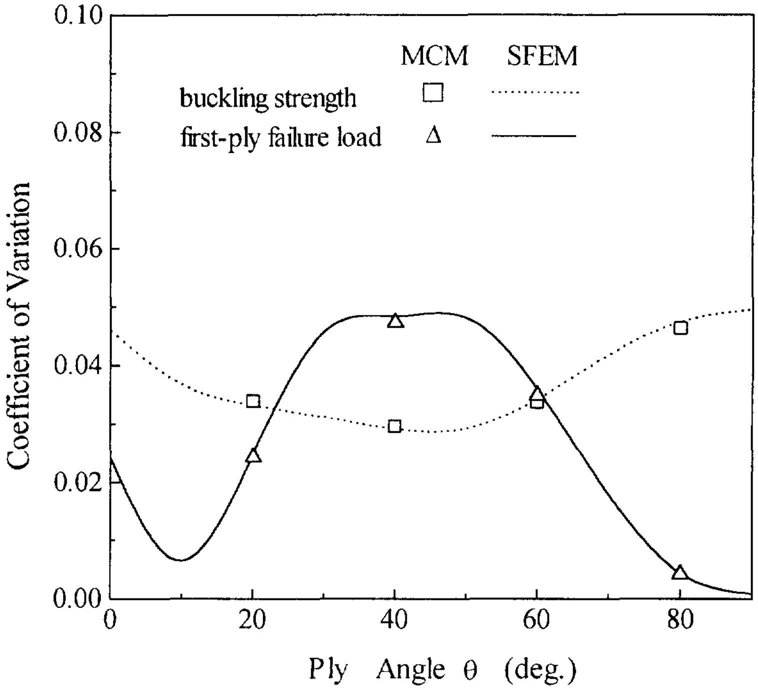

Fig. 8. Coefficients of variation of different types of strength vs fiber angle of a two-layered $[-\Theta/\Theta]$ laminate with random material properties.

angle θ, at less than about 40°. Irrespective of the number of ply groups, the primary failure mode of the laminate is buckling for $\theta < 40°$ if a deterministic approach is adopted. The present SFEM and Monte-Carlo method (MCM) are used to study the coefficient of variation of laminate strength. Figures 7 and 8 show the coefficients of variation for the buckling and first-ply failure load of a $[-\theta/\theta]$ laminate composed of different random parameters. It is noted that the results obtained by the present SFEM closely match those obtained by the MCM in which over 1000 data have been generated for each case. The randomness of layer thickness has greater effects on the variation of laminate buckling

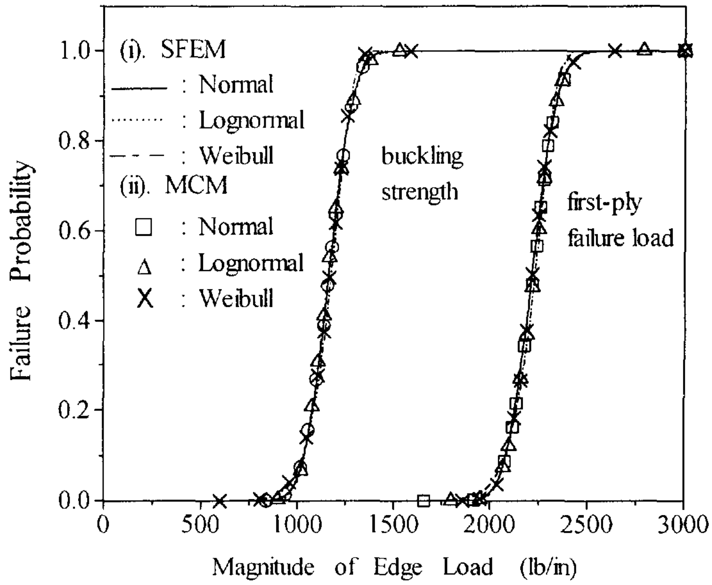

Fig. 9. Failure probabilities of $[-20°/20°/-20°/20°]$ laminate with random material properties of different probability distribution types (C.O.V. is 10%).

strength than that of first-ply failure load, as shown in Fig. 7. When the laminate is composed of random materials, a fiber angle of around 45° may yield the largest variation for buckling load and the smallest for first-ply failure load as shown in Fig. 8.

Next, consider the effects of the probability distribution types of strength ratio or buckling load on failure probability of composite laminates. Herein, the two failure modes, i.e. buckling and first-ply failure are assumed to be independent. The expected fiber angle is set as either $\theta = 20°$ or $\theta = 60°$ for the $[-\theta/\theta/-\theta/\theta]$ laminate. The statistics of the random variables listed in Table 1 are again used in the reliability analysis. The Monte-Carlo method is first used to simulate the failure probabilities of the laminates composed of random material properties, layer thickness or fiber angles and subjected to an edge load of different magnitudes. The random variables are assumed to be of normal, lognormal or Weibull distributions. Random number generators, RNMVN for normal or lognormal variates and RNWIB for Weibull variates in IMSL mathematical package (1989), are adopted in the Monte-Carlo simulation of the laminates. Over 1000 sets of simulation are generated for normal or lognormal variates and 4000 for Weibull variates. An independence check is performed for simulation involved with Weibull variates. The failure probabilities of the laminates are also evaluated via the stochastic finite element method on the basis of either eqn (11) or eqn (12) in which probability density functions of the strength ratio or buckling load are assumed to be normal, lognormal or Weibull distributions. The failure probabilities obtained via the above two approaches are shown in Figs 9–15 for comparison. Figures 9 and 10 show the failure probabilities of the $[-20°/20°/-20°/20°]$ laminate with random material properties of different probability distributions and coefficients of variation (C.O.V.). It is noted that the failure probabilities obtained via the stochastic finite element method closely match those via the Monte-Carlo method. The big gap between the two failure probability curves in Figs 9 or 10 indicates that buckling is the major failure mode of the laminate. For each failure mode, the clustering of the failure probability curves predicted using different probability distribution types for the random material properties, strength ratio or buckling strength indicates that the effects of the types of probability distributions on the failure probabilities of the laminate are small. When comparing the curves in Fig.9 with those in Fig. 10, it is noted that the increase in coefficients of variation of material properties reduces the slopes of the failure probability curves. Similar phenomenon can also be observed for the laminate composed of random layer thicknesses or fiber angles as shown in Figs 11 and 12. Figures 13–15 show the failure probabilities of the $[-60°/60°/-60°/60°]$ laminate composed of various random parameters. Again, the failure

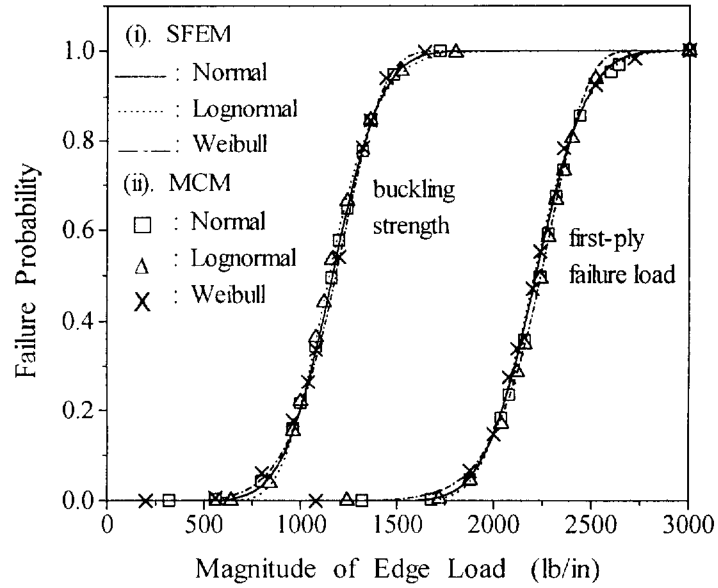

Fig. 10. Failure probabilities of [$-20°/20°/-20°/20°$] laminate with random material properties of different probability distribution types (C.O.V. is 20%).

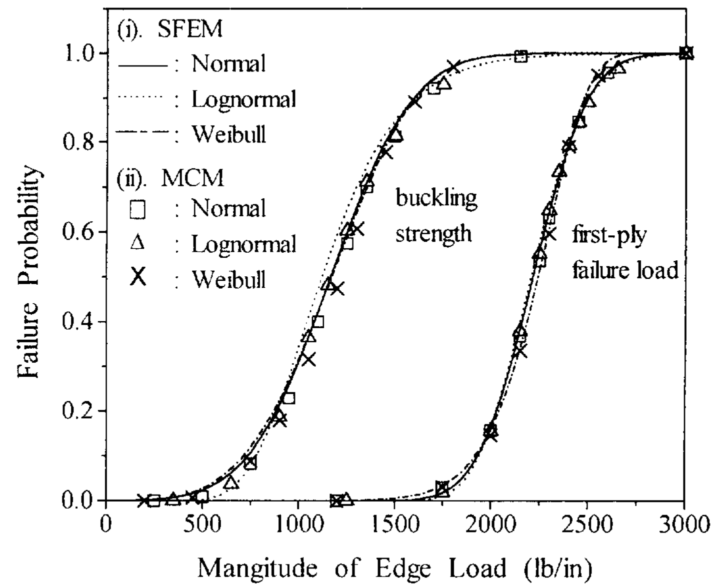

Fig. 11. Failure probabilities of [$-20°/20°/-20°/20°$] laminate with random layer thicknesses of different probability distribution types (C.O.V. is 10%).

probabilities predicted by MCM closely match those predicted by the SFEM and the probability distribution types of the basic random parameters, strength ratio or buckling load have small effects on the failure probabilities of the laminate. It is also noted that in contrast to Figs 9, 11 and 12 the gaps between the buckling and first-ply failure probability curves in Figs 13 and 15 are small and the curves of different failure modes in Fig. 14 even cross over each other. This implies that both buckling and first-ply failure modes are important for the laminates. The shapes of the curves also indicate that the variation of buckling load is much larger than that of the first-ply failure load. Therefore, when considering layer thicknesses as random variables (see Fig. 14), buckling dominates the failure of the laminate for small load, but as load increases first-ply failure becomes more dominant. In conclusion, the results presented in Figs 9–15 show that among the random parameters the randomness of layer thickness has the greatest effect on the failure probability of the angle ply laminates, probability distribution types of basic random parameters and laminate

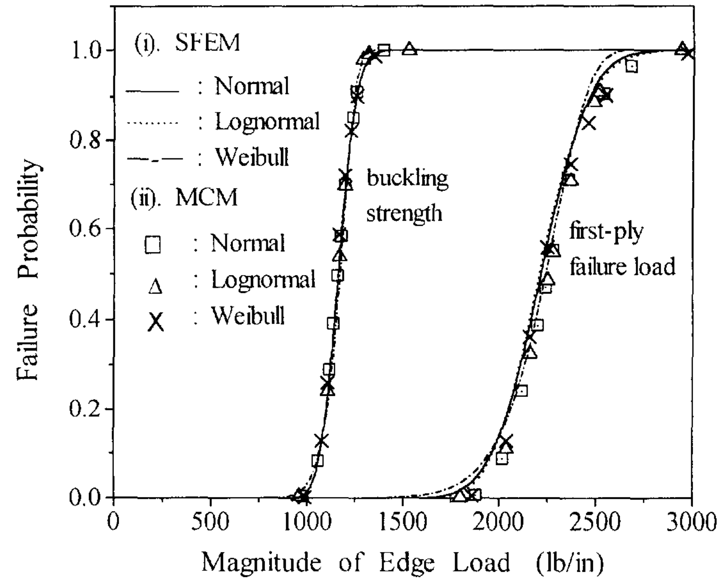

Fig. 12. Failure probabilities of $[-20°/20°/-20°/20°]$ laminate with random fiber angles of different probability distribution types (standard deviation is 3°).

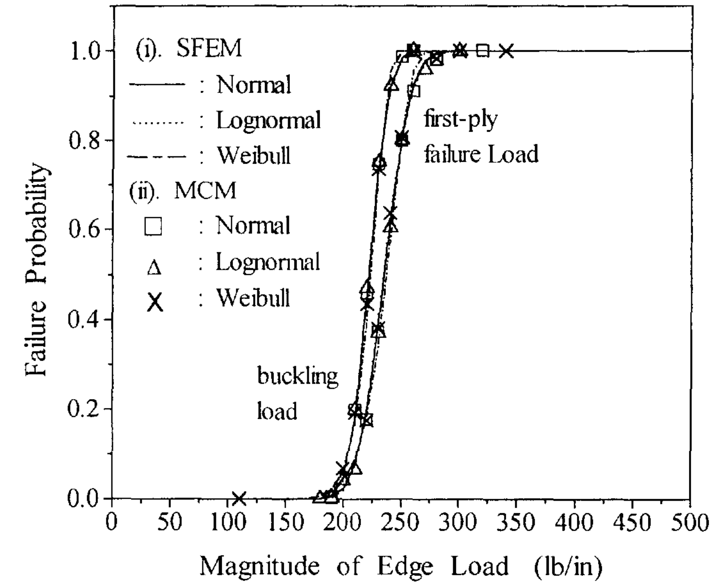

Fig. 13. Failure probabilities of $[-60°/60°/-60°/60°]$ laminate with random material properties of different probability distribution types (C.O.V. is 10%).

strengths have small effects on the failure probability of the laminates, and for small fiber angle buckling dominates the failure of the laminates.

Finally, consider the reliability of angle-ply laminates with same thickness, but composed of different numbers of layer groups. The laminates are subjected to a uniform compressive edge load of 57 lb in^{-1} in X-direction. The reliabilities of the laminates with various random parameters considering either single or multiple failure modes are shown in Figs 16–19. Figures 16–18 show the reliabilities of the laminates considering either buckling or first-ply failure. Figure 19 shows the reliabilities of the laminates with various random parameters considering both buckling and first-ply failure. It is noted that buckling dominates the reliability of the laminates irrespective of the type of random parameters and number of layer groups considered in the analysis because the variation of buckling strength is larger than that of first-ply failure load and the applied load is small. In general, the increase in the number of layer groups will decrease the buckling failure probability,

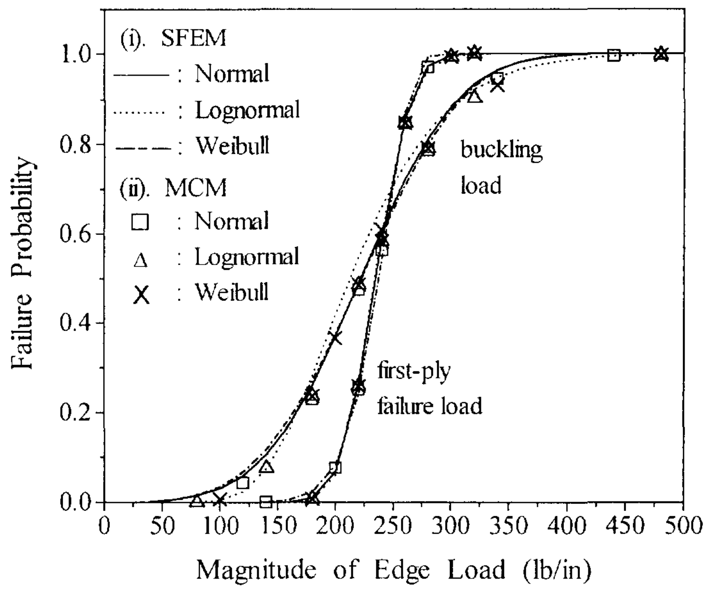

Fig. 14. Failure probabilities of [$-60°/60°/-60°/60°$] laminate with random layer thicknesses of different probability distribution types (C.O.V. is 10%).

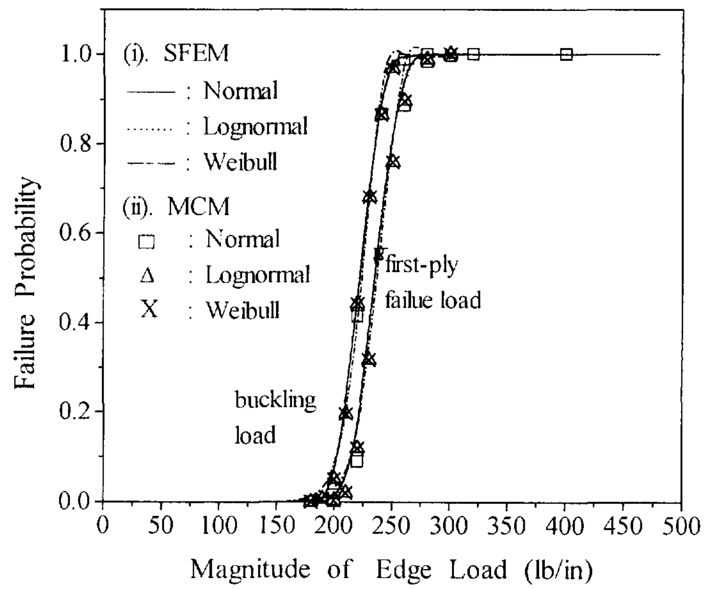

Fig. 15. Failure probabilities of [$-60°/60°/-60°/60°$] laminate with random fiber angles of different probability distribution types (standard deviation is 3°).

but, on the other hand, increase the first-ply failure probability of the laminate. Figure 20 shows the correlation between buckling and first-ply failure loads of a [$-\theta/\theta$] laminate. The results obtained by the SFEM closely match those obtained by the MCM. Buckling load is perfectly correlated with first-ply failure load when only random layer thicknesses are considered in the analysis. Buckling load may be negatively correlated with first-ply failure load if fiber angles or material properties are random and mean fiber angles are large (e.g. $\theta > 40°$ for random material properties).

9. CONCLUSIONS

A procedure for reliability analysis of composite laminates with single or multiple failure modes has been developed on the basis of the stochastic finite element method. The accuracy of the stochastic finite element method in predicting statistics of buckling and

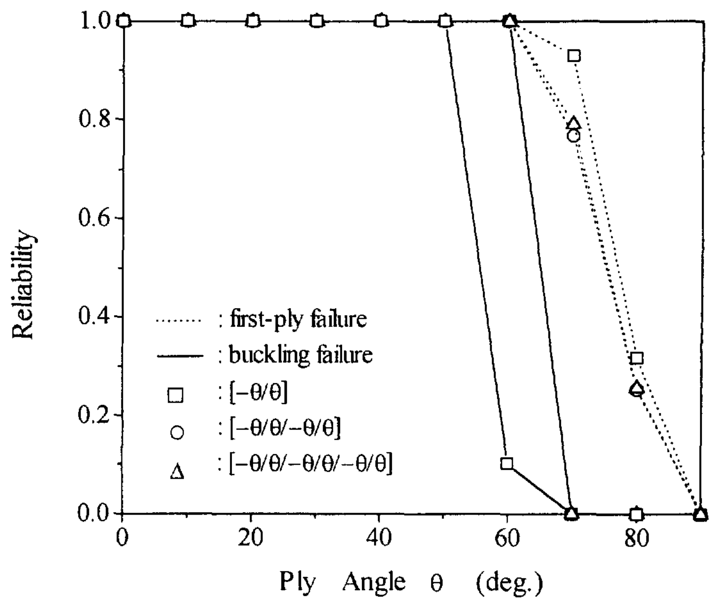

Fig. 16. Reliabilities of laminates with random fiber angles considering either buckling or first-ply failure.

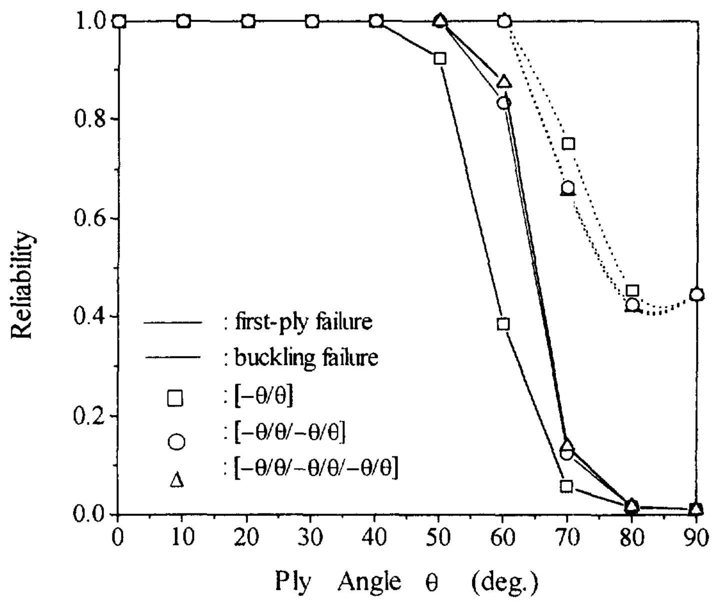

Fig. 17. Reliabilities of laminates with random layer thicknesses considering either buckling or first-ply failure.

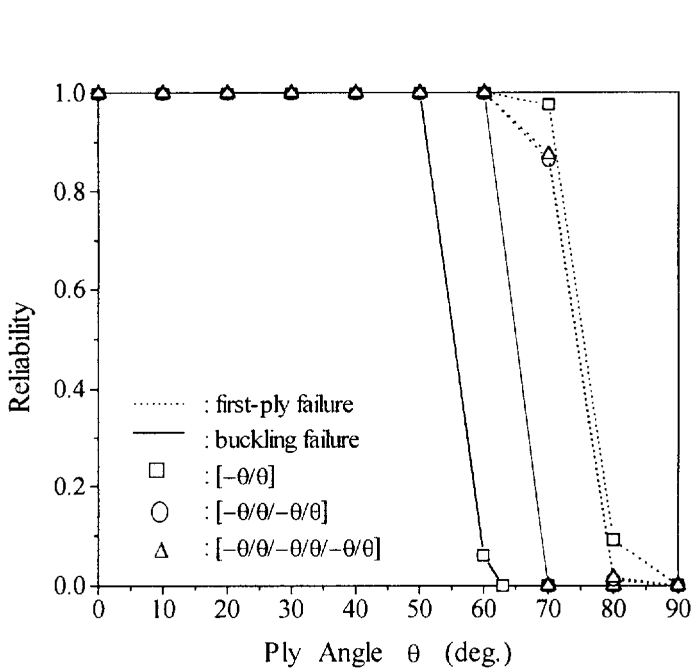

Fig. 18. Reliabilities of laminates with random material properties considering either buckling or first-ply failure.

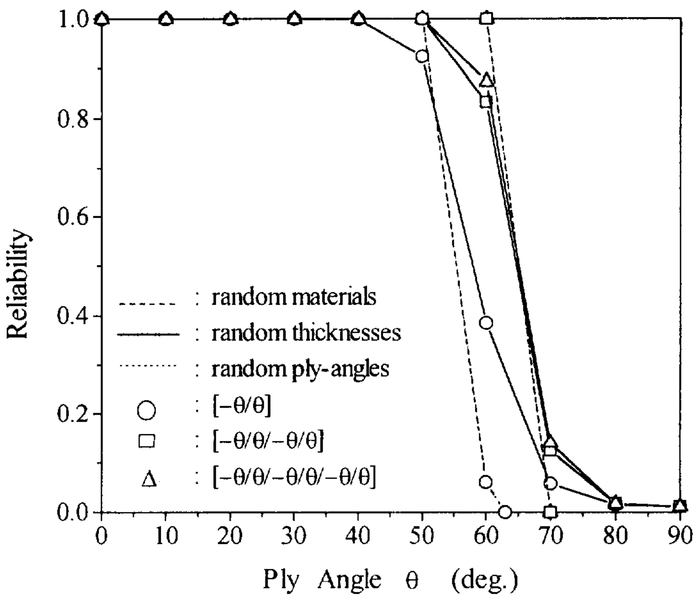

Fig. 19. Reliabilities of laminates with various random parameters considering both buckling and first-ply failure.

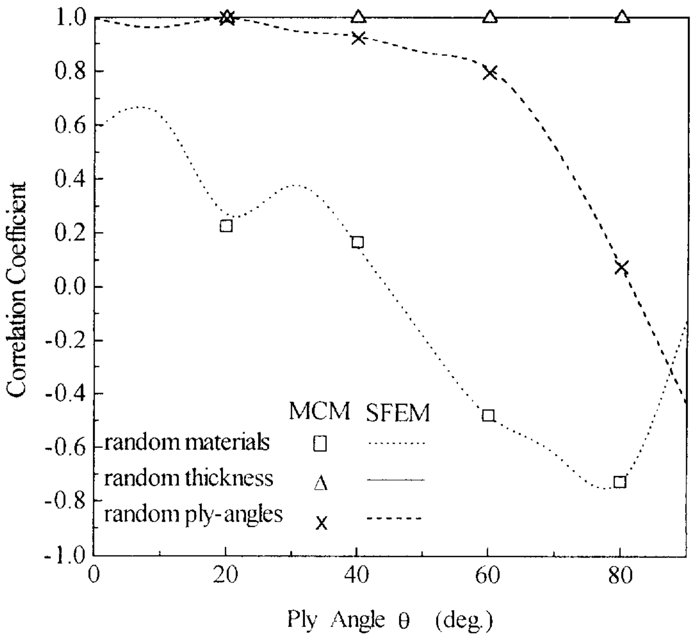

Fig. 20. Correlation between buckling strength and first-ply failure load of two-layered [Θ/−Θ] laminate with different fiber angles and various types of uncertainties.

first-ply failure loads has been verified by the Monte-Carlo method. The applications of the proposed procedure have been demonstrated by means of the reliability predictions of angle-ply laminates with different types of failure modes subject to in-plane edge loads. It has been shown that the variations of ply thicknesses have the greatest effects on the variations of laminate strengths as well as laminate reliability. Thus, tight control on ply thickness variation is essential for achieving high reliability. Other important random variables, such as applied loads, initial imperfections and lamina strengths, which were not included in the present method should be considered in future studies.

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